tained, interpretation has been incorrect. A further test is that the same sites with the same occupancies should be found in different centrosymmetric projections of any one adduct.

Similar procedures could be used for other centrosymmetric plane groups. The statement that in an $A B$ correlation map an $A$ site is at the centre of a group of vector peaks representing the $B$ sites, and vice versa, is valid wherever there is a centre of symmetry; but the multiplicity and orientation of the group of peaks for any one site depend on the plane group symmetry. For instance, for the one centrosymmetric projection of monoclinic protein crystals (symmetry $p 2$ ), there would be, around each $A$ site, pairs of diametrically opposite peaks representing $B$ sites, and the diameters would be in different radial directions, not parallel to any axis; and in plane group $p 4$ (for the $c$ projection of some tetragonal crystals), the groups are squares with their sides inclined to the cell axes. A peak at the origin is always an indication of a common site, and the superposition procedures, for Patterson maps on a correlation map or for different correlation maps on each other, are the same as those given for the orthorhombic example.

There are obviously limitations to the complexity of the structures that might be solved by these methods. The number of sites, multiplicity of equivalent positions, range of occupancies and the effective resolution of the data employed are all involved:
(i) For the twofold multiplicity of a monoclinic
centric projection, it should be possible to deal with a larger number of sites than for the fourfold orthorhombic multiplicity.
(ii) It will probably be possible to locate only the sites of relatively high occupancy; sites of low occupancy would have to be detected by established methods based on electron density maps phased by the sites of high occupancy.

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# The Crystal Structure of $\mathbf{U Z n}_{12}{ }^{*}$ 

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#### Abstract

The crystal structure of the most zinc-rich compound in the uranium-zinc system is hexagonal with space group symmetry $P 6 / \mathrm{mmm}$. Crystals formed by cooling a $95 \mathrm{wt} . \%$ zinc alloy have a composition range of $\mathrm{UZn}(10 \cdot 4-11 \cdot 0) \pm 0 \cdot 1$ as determined by microprobe analysis; lattice parameters are $a=8.950 \pm$ 0.001 and $c=8.902 \pm 0.002 \AA$. The structure is based on an ideal $\mathrm{UZn}_{12}$ stoichiometry, but substitution between pairs of zinc atoms and uranium atoms leads to lower concentrations of zinc, $\mathrm{UZn}_{10 \cdot 4}$ being found for the crystal studied. The reported equivalence of the $\mathrm{SmZn}_{12}$ and $\mathrm{UZn}_{12}$ structures based on powder work is confirmed. They differ only in the extent of substitution at different lattice sites.


## Introduction

Two intermetallic compounds are known to exist in the uranium-zinc system. $\mathrm{U}_{2} \mathrm{Zn}_{17}$ is reported to have both a rhombohedral and a hexagonal form. The status of the structural work has been summarized by Johnson, Smith \& Wood (1968).

[^0]The existence of a second compound richer in zinc than $\mathrm{U}_{2} \mathrm{Zn}_{17}$ has been shown by a series of investigations by researchers at the Argonne National Laboratory (Martin \& Wach, 1960, $1962 a$ \& 1962b; Velekis \& Goetzinger, 1960; Argonne National Laboratory, 1965). They report a hexagonal compound with a composition range of $\mathrm{UZn}_{9 \cdot 36}$ to $\mathrm{UZn}_{11 \cdot 47}$. Veleckis, Schablaske \& Tani (1966), using power methods, found that this compound, designated $\mathrm{UZn}_{12}$, is isostructural with the high-temperature form of $\mathrm{SmZn}_{12}$. A detailed
single-crystal determination of the high-temperature form of $\mathrm{SmZn}_{12}$ by Mason, Harsha \& Chiotti (1970) has shown that substitution between pairs of zinc atoms and samarium atoms results in a structure with an approximate composition of $\mathrm{Sm} \mathrm{Zn}_{11}$. The purpose of this work is to determine the structure of the zincrich uranium-zinc compound.

## Experimental procedure

The materials used were reactor grade uranium of $99.9 \%$ purity and slab zinc of $99.99+\%$ purity from the Bunker Hill Co. A 95 wt. \% zinc alloy was sealed under argon in a tantalum crucible which in turn was enclosed in stainless steel. The alloy was equilibrated at $900^{\circ} \mathrm{C}$ and slowly cooled to precipitate the zinc-rich compound. The formed crystallites were isolated by selectively dissolving the zinc matrix with diluted nitric acid. Because of the difficulty in obtaining homogeneous bulk samples, density measurements and chemical analyses were not made.

Two samples were examined by an electron microprobe: a uranium-zinc diffusion couple and a portion of the $95 \mathrm{wt} . \%$ zinc alloy. The $\mathrm{UZn}_{8.5}$ compound was used as a reference standard in order to minimize the absorption and fluorescence effects on the measurements of the unknown compound. The measured compositions varied from $\mathrm{UZn}_{10.4}$ to $\mathrm{UZn}_{11.0}$ for crystals in both samples, with an indicated precision in the zinc-to-uranium ratio of $\pm 0 \cdot 1$. The total accuracy of the measurements, however, is not known.

Lattice parameters were determined from DebyeScherrer powder patterns of crystallites from the 95 wt. \% alloy, using nickel-filtered copper radiation ( $\lambda=$ $1.5418 \AA$ ). A least-squares procedure, utilizing the Nel-son-Riley extrapolation function, gave lattice parameters of $a=8.950 \pm 0.001$ and $c=8.902 \pm 0.002 \AA$.

All single crystals examined by oscillation and Weissenberg methods had the same Laue symmetry, $6 / \mathrm{mmm}$. The observed Weissenberg photographs were essentially identical in spot position and intensity with the corresponding $\mathrm{Sm}_{\mathrm{Zn}}^{12}$ films. No systematic extinctions were observed. The possible space groups are $P \overline{6} m 2$,
$P 6 \mathrm{~mm}, P 622, P \overline{6} 2 \mathrm{~m}$ and $P 6 / \mathrm{mmm}$. Relative intensity data were obtained by the $2 \theta$ scan technique using a General Electric single-crystal orienter equipped with a scintillation counter. Zirconium-filtered Mo $K \alpha$ radiation $(\lambda=0.7107 \AA)$ was used. All reflections of the type $h k l$ with $h \geq k$ were examined in the range $0-70^{\circ}, 2 \theta$. The $\varphi$ dependence was measured at $\chi=90^{\circ}$.

The crystal used was basically a prism of height 0.20 mm and maximum thickness 0.09 mm . Several edges were cleaved, making the crystal rather irregular. The measured $\varphi$ dependence was used to determine a $\varphi$-dependent absorption correction for all reflections. Reflections were considered observed if ( $I$-background) $>2 \cdot 5(I+\text { background })^{1 / 2}$. Of the 599 possible reflections, 345 were observed. Atomic scattering factors were taken from Cromer \& Waber (1965). Anomalous dispersion terms were taken from Cromer (1965). The full-matrix least-squares program $O R F L S$ of Busing, Martin \& Levy (1962) was used in the refinement. All $R$ indices given are for $\sum\left|F_{o}-K F_{c}\right| / \sum\left|F_{o}\right|$ where $K$ is a scale factor and only observed reflections are included.

## Structure determination

The initial trial model was based on the centric space group $P 6 / \mathrm{mmm}$ with the $\mathrm{Sm} \mathrm{Zn}_{12}$ parameters as input. This gave an $R$ index of $0 \cdot 16$, indicating the essential correctness of the model. The strong low-angle reflections showed extinction effects and the strongest were excluded from the initial refinements. A series of difference Fourier analyses and least-squares refinements indicated a substitution of uranium atoms for pairs of zinc atoms. The uranium atoms occupy the $\frac{1}{3}, \frac{2}{3}, 0$ positions $24 \%$ of the time, with a corresponding decrease to $76 \%$ of the zinc pair occupancy of the $\frac{1}{3}, \frac{2}{3}, 0 \cdot 14$ positions. The criterion used to obtain the degree of substitution about any position was the minimization of the difference Fourier. The $R$ index based on this defect model and using anisotropic temperature factors was $0 \cdot 055$. The $R$ index including the strong low-angle reflections was 0.074 . Extinction corrections of the form $F_{o}($ corrected $)=F_{o}\left(1+\beta I_{o}\right)($ Zachariasen, 1963), where $\beta$ is the extinction parameter and $I_{o}$ is the measured

Table 1. Least-squares parameters for $\mathrm{UZn}_{12}$ (space group P6/mmm)

|  | Wyckoff notation | $x$ | $y$ | $z$ | $\begin{aligned} & \beta_{11}^{*} \\ & \times 10^{4} \end{aligned}$ | $\begin{gathered} \beta_{22} \\ \times 10^{4} \end{gathered}$ | $\begin{gathered} \beta_{33} \\ \times 10^{4} \end{gathered}$ | $\begin{aligned} & \beta_{12} \\ & \times 10^{4} \end{aligned}$ | $\begin{aligned} & \beta_{13} \\ & \times 10^{4} \end{aligned}$ | $\begin{aligned} & \beta_{23} \\ & \times 10^{4} \end{aligned}$ | Occupancy |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Zn}(1)$ | 12 (o) | $0 \cdot 1683$ (2) | $2 x$ | $0 \cdot 2390$ (4) | 33 (2) | 51 (3) | 51 (4) | ${ }^{\frac{1}{2}} \beta_{22}$ | 1 (1) | $2 \beta_{13}$ | 100 |
| $\mathrm{Zn}(2)$ | 6 (j) | $0 \cdot 3550$ (6) | $0 \cdot 0$ | 0.0 | 102 (7) | 281 (18) | 42 (7) | $\frac{1}{2} \beta_{22}$ |  | $2 \beta_{13}$ | 100 |
| $\mathrm{Zn}(3)$ | 6 (k) | $0 \cdot 2953$ (5) | $0 \cdot 0$ | $\frac{1}{2}$ | 38 (4) | 19 (5) | 36 (6) | $\frac{1}{2} \beta_{22}$ | - - | - - | 100 |
| $\mathrm{Zn}(4)$ | 6 (i) | $\frac{1}{2}$ | $0 \cdot 0$ | $0 \cdot 2753$ (6) | 44 (4) | 54 (7) | 60 (5) | ${ }_{\frac{1}{2}} \beta_{22}$ | - - | - - | 100 |
| Zn (5) | 2 (e) | $0 \cdot 0$ | $0 \cdot 0$ | $0 \cdot 3537$ (7) | 37 (6) | $\beta_{11}$ | 13 (6) | ${ }_{1}^{1} \beta_{22}$ | - - | - - | 100 |
| $\mathrm{Zn}(6)$ | 4 (h) | $\frac{1}{3}$ | $\frac{2}{3}$ | $0 \cdot 1472$ (10) | 72 (8) | $\beta_{11}$ | 56 (9) | ${ }_{1}^{2} \beta_{22}$ | - - | - - | 76 |
| $\mathbf{Z n}(7) \dagger$ | 4 (h) | $\frac{1}{3}$ | $\frac{2}{3}$ | $0 \cdot 3492$ (135) | 71 | $\beta_{11}$ | 64 | $\frac{1}{2} \beta_{22}$ | - - | - - | 5 |
| U(1) | 1 (a) | $0 \cdot 0$ | $0 \cdot 0$ | $0 \cdot 0$ | 40 (3) | $\beta_{11}$ | 28 (4) | $\frac{1}{2} \beta_{22}$ | - - | - - | 100 |
| U(2) | 2 (d) | $\frac{1}{3}$ | 2 | $\frac{1}{3}$ | 18 (2) | $\beta_{11}$ | 24 (3) | ${ }_{1} \beta_{22}$ | - - | - - | 95 |
| U(3) | 2 (c) | $\frac{1}{3}$ | $\frac{2}{3}$ | $0 \cdot 0$ | 40 (8) | $\beta_{11}$ | 42 (10) | $\frac{1}{2} \beta_{22}$ | - - | - - | 24 |

[^1]intensity, were applied to the data. A value of $\beta=$ $1.0 \times 10^{-4}$ gave the lowest $R$ index ( $0 \cdot 050$ ) after two refinement cycles using all observed reflections.

Table 2. Observed and calculated structure factors

The difference Fourier at this stage indicated that a partial substitution of the uranium atoms at the $\frac{1}{3}, \frac{2}{3}, \frac{1}{2}$ positions by pairs of zinc atoms at $\frac{1}{3}, \frac{2}{3}, 0 \cdot 35$ was also occurring $5 \%$ of the time. Inclusion of this substitution lowered the $R$ index to 0.048 and resulted in a composition of $\mathrm{UZn}_{10 \cdot 4}$.

Several different weighting schemes were tried during the course of the structure determination, but none appeared significantly better than another. Final parameters (Table 1) were obtained using the extinction corrected data and unit weights. The standard deviations given are those calculated by the least-squares refinement program. Observed and calculated structure factors are given in Table 2.

## Discussion

The basic structure of the zinc-rich uranium-zinc compound has a stoichiometry of $\mathrm{UZn}_{12}$. This compound is one of a family of compounds related to the $\mathrm{CaZn}_{5}$, $D 2_{a}$ type structure. Such compounds can be formed by the systematic replacement of the large atoms in the $D 2_{d}$ structure by a pair of small atoms aligned along the $Z$ direction and centered on the large atom site. By using different replacement sequences, stoichiometries of $A_{3} B_{22}, A_{2} B_{17}$, and $A B_{12}$ can be generated. In some compounds, the replacement process is incomplete and the lattice positions are filled partially by large atoms and partially by small atom pairs. This incomplete substitution occurs in the $\mathrm{SmZn}_{12}$ compound, Mason et al. (1970), and in the uranium-zinc compound examined here.

In the $\mathrm{UZn}_{10 \cdot 4}$ structure, $24 \%$ of the $\frac{1}{3}, \frac{2}{3}, 0$ positions are occupied by $U(3)$ atoms and $76 \%$ of the positions are occupied by a pair of $\mathrm{Zn}(6)$ atoms (the center of the pair is at the $\frac{1}{3}, \frac{2}{3}, 0$ position, with the atoms along the $Z$ direction as represented by the positions $\frac{1}{3}, \frac{2}{3}, 0 \cdot 14$ ). This same type of mixed occupancy occurs to a smaller extent at the $\frac{1}{3}, \frac{2}{3}, \frac{1}{2}$ position involving $\mathrm{U}(2)$ and $\mathrm{Zn}(7)$ atoms. No substitution of uranium at the position $0,0, \frac{1}{2}$ was observed in contrast with the $\mathrm{SmZn}_{12}$ structure. Interatomic distances out to $4 \AA$ are listed in Table 3. The greatest contraction among atoms not involved in the substitutions is $7.0 \%$ between $\mathrm{Zn}(1)$ and $\mathrm{Zn}(4)$. Coordination polyhedra about the uranium atoms are identical to those described in the $\mathrm{SmZn}_{12}$ structure.

Table 3. Interatomic distances

|  | Type <br> neighbor | Number of <br> neighbors | Distancr <br> $\AA$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{U ( 1 )}$ | $\mathrm{Zn}(5)$ | 2 | $3 \cdot 151$ |
|  | $\mathrm{Zn}(2)$ | 6 | $3 \cdot 178$ |
| $\mathbf{U ( 2 )}$ | $\mathrm{Zn}(1)$ | 12 | $3 \cdot 368$ |
|  | $\mathrm{Zn}(6)$ | 2 | $3 \cdot 142$ |
|  | $\mathrm{Zn}(3)$ | 6 | $3 \cdot 168$ |
|  | $\mathrm{Zn}(4)$ | 6 | 3.269 |
| $\mathbf{U ( 3 )}$ | $\mathrm{Zn}(1)$ | 6 | $3 \cdot 458$ |
|  | $\mathrm{Zn}(2)$ | 6 | $2 \cdot 892$ |
|  | $\mathrm{Zn}(7)$ | 2 | $3 \cdot 104$ |

Table 3 (cont.)

$\mathrm{U}(3)-\mathrm{Zn}(6), \mathrm{U}(2)-\mathrm{Zn}(7)$ and $\mathrm{Zn}(6)-\mathrm{Zn}(7)$ pairs do not occur simultaneously.

Microprobe results on crystallites in the $95 \mathrm{wt} . \%$ zinc alloy indicate a composition range of $\mathrm{UZ}_{10 \cdot 4}$ to $\mathrm{UZn}_{11 \cdot 0}$. Lattice parameters obtained from powdered crystallites extracted from the same alloy indicate a composition of $\mathrm{UZn}_{11 \cdot 0}$. This composition is based of
the assumption of a linear relation between the composition limits reported by Veleckis \& Goetzinger (1960) and on the lattice parameters reported by workers at the Argonne National Laboratory (1965). The observed composition range indicates a solidus curve, for the zinc-rich alloy, which varies with temperature.

There was no evidence of a low-temperature modification of the $\mathrm{UZn}_{12}$ compound corresponding to the tetragonal $\mathrm{Sm}_{\mathrm{Zn}}^{12}$ form: All examined crystals were hexagonal. However, no special effort was made to prepare a low-temperature form; therefore, such a structure cannot be ruled out.

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[^1]:    $*$ The anisotropic temperature-factor relation used is $\exp \left[-\left(h^{2} \beta_{11}+k^{2} \beta_{22}+l^{2} \beta_{33}+2 h k \beta_{12}+2 h l \beta_{13}+2 k l \beta_{23}\right)\right]$.
    $\dagger$ Temperature terms held constant during refinement cycle.

